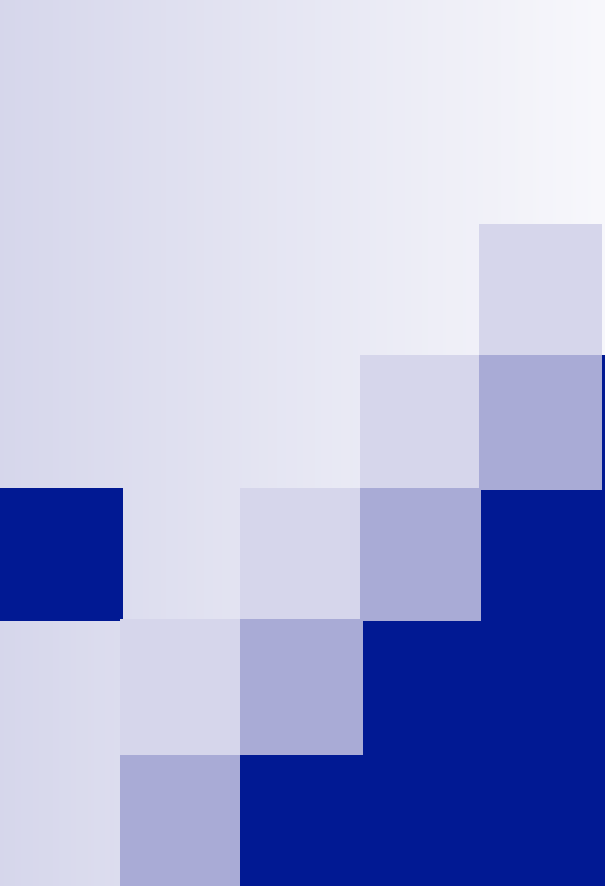




# **Virtual Memory and Paging (1)**

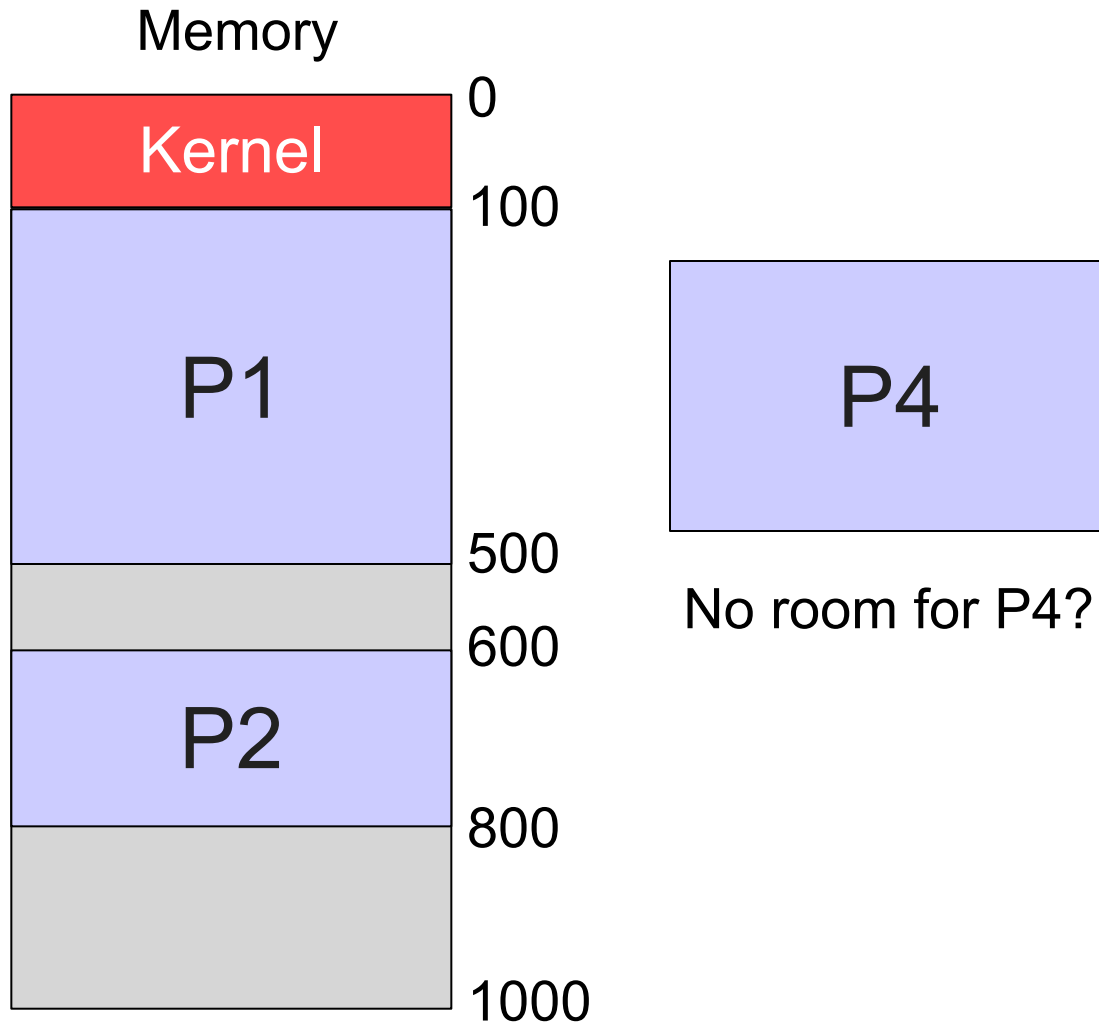
**ICS332  
Operating Systems**

Henri Casanova ([henric@hawaii.edu](mailto:henric@hawaii.edu))

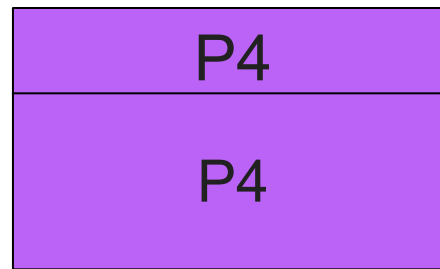
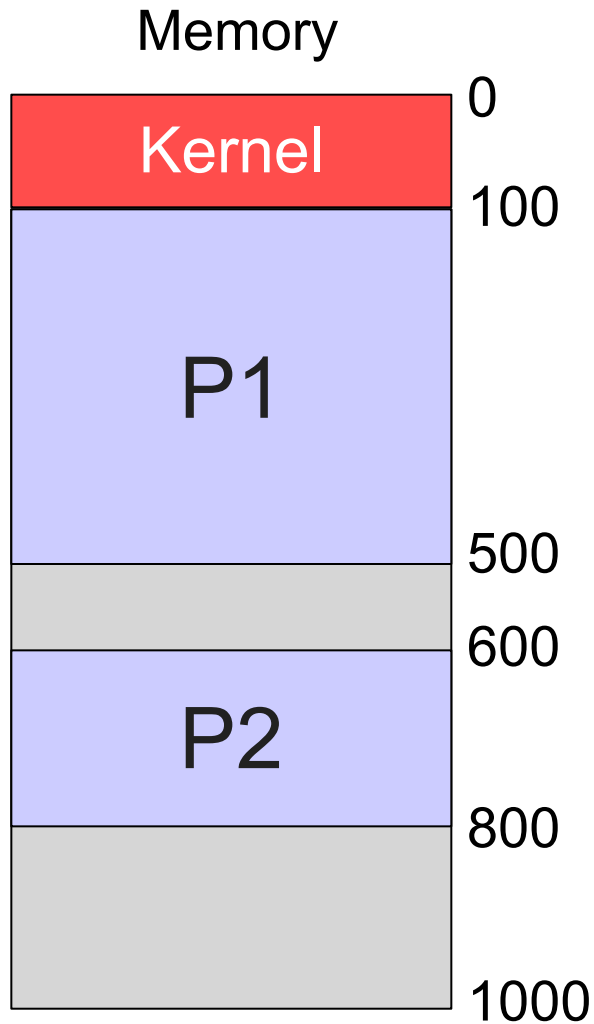
# Conclusion (Previous Module)

- **Assumption so far:** Each process is in a **contiguous** address space
  - I'll assume a single segment, for simplicity ("address space" = "segment" in these lecture notes)
- 😊 Address virtualization is simple
  - Just a base register and a limit register, a comparison, an addition
- 😬 No "best" memory allocation strategies
  - First Fit, Worst Fit, Best Fit, others??
- 😡 Fragmentation can be very large
  - RAM is wasted
- 😡 There can be process starvation in spite of sufficient available RAM due to fragmentation
  - 100 1MiB holes don't allow a 100MiB process to run!
- **Conclusion:** Our base assumption is flawed!
- So.... address spaces shouldn't be contiguous!?!

# Non-Contiguous Address Space

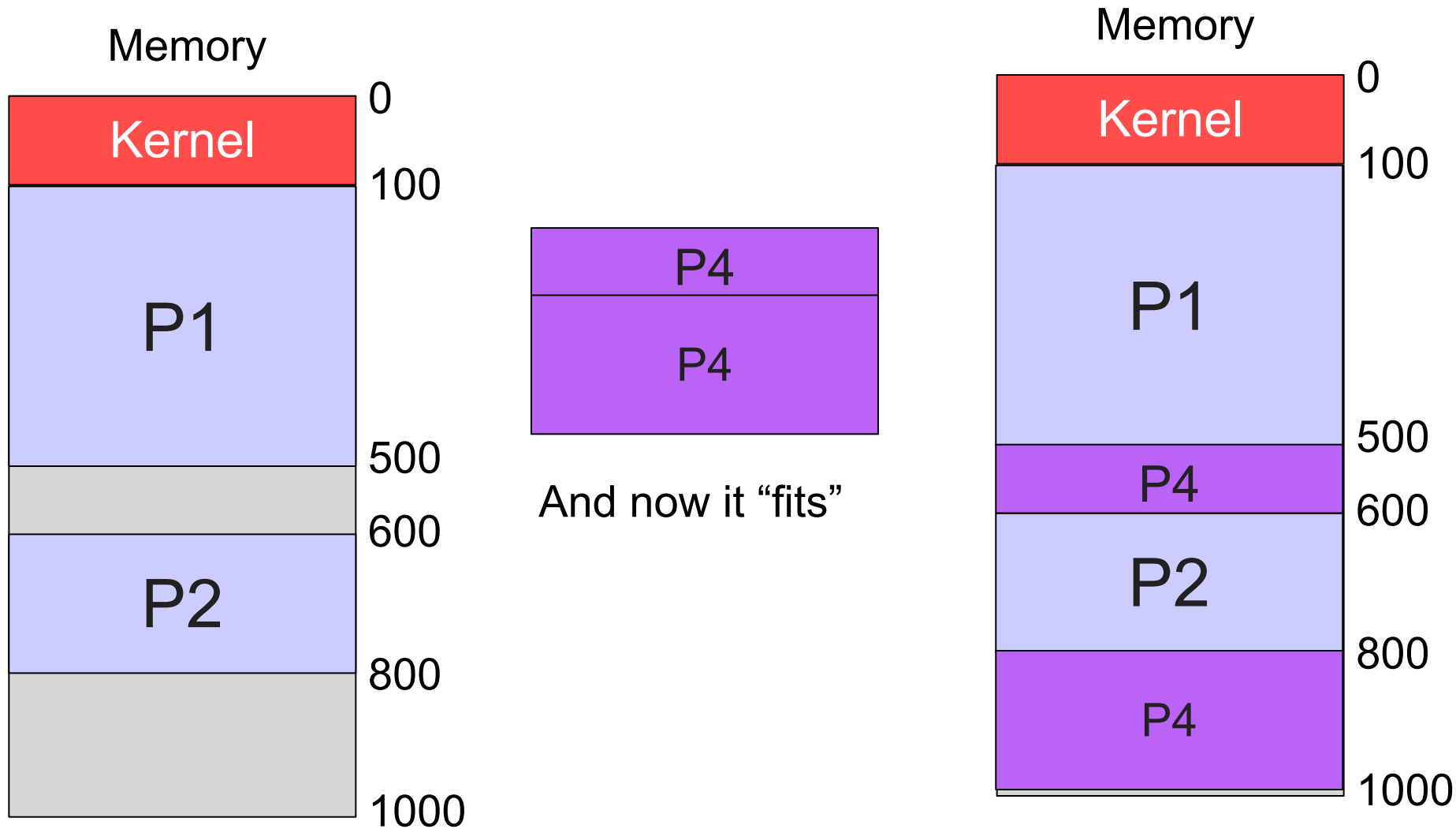


# Non-Contiguous Address Space



Let's "chop it up"

# Non-Contiguous Address Space



# The Solution: “Paging”

- Our solution: break up address spaces into smaller chunks
- Should we have chunks of variable size like we just did on the previous example?
- Not a good idea as this is a well-known difficult problem algorithmically: Bin Packing
  - Known to be NP-hard
- We really don't want for the OS to have to solve some NP-hard problem
- **But if chunk sizes are fixed, it all becomes easy!**
  - No longer NP-hard
- So that's what we do!
- Each process' address spaces in split into same-size “pages”
- This is called **Paging**

# Paging

- The physical memory is split in fixed-size **frames**, and **each frame can hold a page** (frame size = page size)
- A page is “virtual” (or “logical”): Virtual Page Number (VPN)
- A frame is physical: Physical Frame Number (PFN)

|    |  |
|----|--|
| 0  |  |
| 1  |  |
| 2  |  |
| 3  |  |
| 4  |  |
| 5  |  |
| 6  |  |
| 7  |  |
| 8  |  |
| 9  |  |
| 10 |  |
| 11 |  |
| 12 |  |
| 13 |  |
| 14 |  |
| 15 |  |

# Paging

- The physical memory is split in fixed-size **frames**, and **each frame can hold a page** (frame size = page size)
- A page is “virtual” (or “logical”): Virtual Page Number (VPN)
- A frame is physical: Physical Frame Number (PFN)

|    |        |
|----|--------|
| 0  | Kernel |
| 1  | Kernel |
| 2  |        |
| 3  |        |
| 4  |        |
| 5  |        |
| 6  |        |
| 7  |        |
| 8  |        |
| 9  |        |
| 10 |        |
| 11 |        |
| 12 |        |
| 13 |        |
| 14 |        |
| 15 |        |



# Paging

- The physical memory is split in fixed-size **frames**, and **each frame can hold a page** (frame size = page size)
- A page is “virtual” (or “logical”): Virtual Page Number (VPN)
- A frame is physical: Physical Frame Number (PFN)

**P1's LOGICAL address space**

|             |
|-------------|
| P1 - page 0 |
| P1 - page 1 |
| P1 - page 2 |
| P1 - page 3 |

|    |        |
|----|--------|
| 0  | Kernel |
| 1  | Kernel |
| 2  |        |
| 3  |        |
| 4  |        |
| 5  |        |
| 6  |        |
| 7  |        |
| 8  |        |
| 9  |        |
| 10 |        |
| 11 |        |
| 12 |        |
| 13 |        |
| 14 |        |
| 15 |        |

# Paging

- The physical memory is split in fixed-size **frames**, and **each frame can hold a page** (frame size = page size)
- A page is “virtual” (or “logical”): Virtual Page Number (VPN)
- A frame is physical: Physical Frame Number (PFN)

P1's LOGICAL address space

|             |
|-------------|
| P1 - page 0 |
| P1 - page 1 |
| P1 - page 2 |
| P1 - page 3 |

|    |             |
|----|-------------|
| 0  | Kernel      |
| 1  | Kernel      |
| 2  | P1 - page 0 |
| 3  |             |
| 4  | P1 - page 3 |
| 5  |             |
| 6  |             |
| 7  |             |
| 8  |             |
| 9  |             |
| 10 |             |
| 11 |             |
| 12 | P1 - page 1 |
| 13 | P1 - page 2 |
| 14 |             |
| 15 |             |

P1's PHYSICAL address space

# Paging

- The physical memory is split in fixed-size **frames**, and **each frame can hold a page** (frame size = page size)
- A page is “virtual” (or “logical”): Virtual Page Number (VPN)
- A frame is physical: Physical Frame Number (PFN)

|    |             |
|----|-------------|
| 0  | Kernel      |
| 1  | Kernel      |
| 2  | P1 - page 0 |
| 3  | P2 - page 2 |
| 4  | P1 - page 3 |
| 5  | P2 - page 1 |
| 6  | P2 - page 0 |
| 7  |             |
| 8  | P2 - page 3 |
| 9  |             |
| 10 | P2 - page 4 |
| 11 |             |
| 12 | P1 - page 1 |
| 13 | P1 - page 2 |
| 14 |             |
| 15 |             |

# Paging

- The physical memory is split in fixed-size **frames**, and **each frame can hold a page** (frame size = page size)
- A page is “virtual” (or “logical”): Virtual Page Number (VPN)
- A frame is physical: Physical Frame Number (PFN)
- And just like that, we have **non-contiguous memory allocation**
- We still have internal fragmentation, but never external fragmentation!

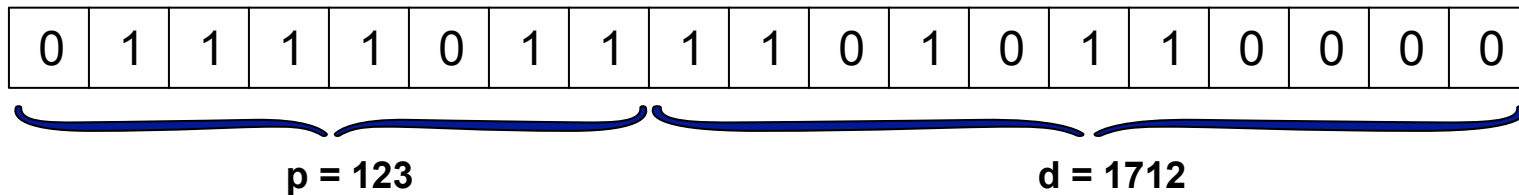
|    |             |
|----|-------------|
| 0  | Kernel      |
| 1  | Kernel      |
| 2  | P1 - page 0 |
| 3  | P2 - page 2 |
| 4  | P1 - page 3 |
| 5  | P2 - page 1 |
| 6  | P2 - page 0 |
| 7  | P3 - page 0 |
| 8  | P2 - page 3 |
| 9  |             |
| 10 | P2 - page 4 |
| 11 | P3 - page 1 |
| 12 | P1 - page 1 |
| 13 | P1 - page 2 |
| 14 | P3 - page 2 |
| 15 |             |

# Paging and Addressing

- In the previous picture you see that a process' address space is non-contiguous and pages are not even in the “right order”
- When we used to say “some byte is at offset X from the beginning of the address space”, now we have to say “some byte is at offset Z from the beginning of the Y-th page of the address space”
- So when we're given a logical address, we have to compute: the **virtual page number** and the **offset within that page**
- For instance, if the page/frame size is 1000 bytes, and we're talking about the 1200-th byte in the address space, then we say that the virtual page number is 1 and the offset is 200!
  - Now you see why we talked about parking lots in the Counting and Addressing module (spots are bytes, blocks of spots are pages)

# Virtual Page number

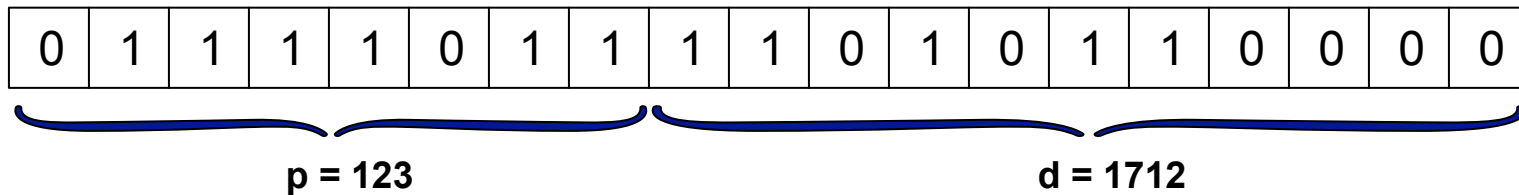
- Virtual addresses issued by the CPU are split into two parts



- The virtual/logical page number:  $p$
- The offset within the page:  $d$
- “Read the value at address  $x$ ” becomes “read the value at offset  $d$  in page  $p$ ”

# Virtual Page number

- Virtual addresses issued by the CPU are split into two parts

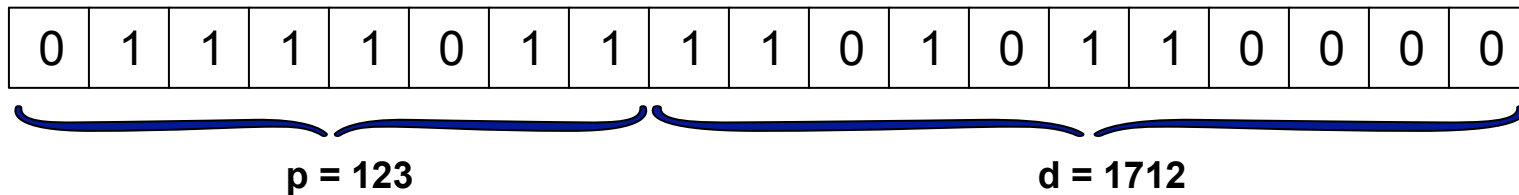


- The virtual/logical page number:  $p$
- The offset within the page:  $d$
- “Read the value at address  $x$ ” becomes “read the value at offset  $d$  in page  $p$ ”

In the above example, how many pages can the process have?

# Virtual Page number

- Virtual addresses issued by the CPU are split into two parts



- The virtual/logical page number:  $p$
- The offset within the page:  $d$
- “Read the value at address  $x$ ” becomes “read the value at offset  $d$  in page  $p$ ”

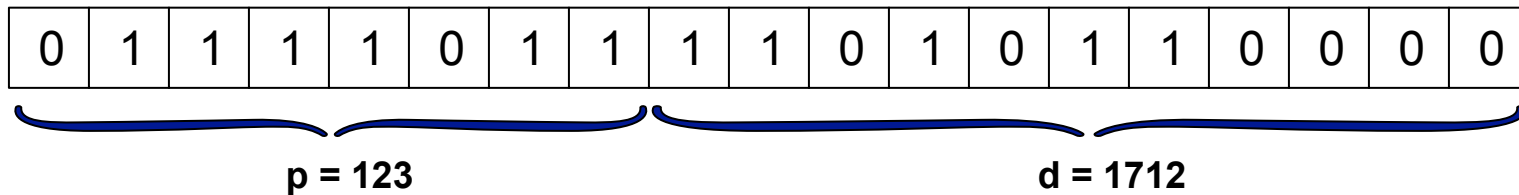
In the above example, how many pages can the process have?

**8 bits  $\rightarrow 2^8 = 256$  pages**



# Virtual Page number

- Virtual addresses issued by the CPU are split into two parts

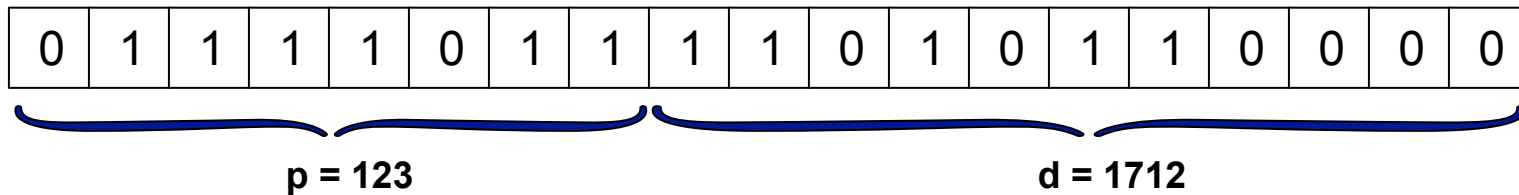


- The virtual/logical page number:  $p$
- The offset within the page:  $d$
- “Read the value at address  $x$ ” becomes “read the value at offset  $d$  in page  $p$ ”

In the above example, how big is each page?

# Virtual Page number

- Virtual addresses issued by the CPU are split into two parts



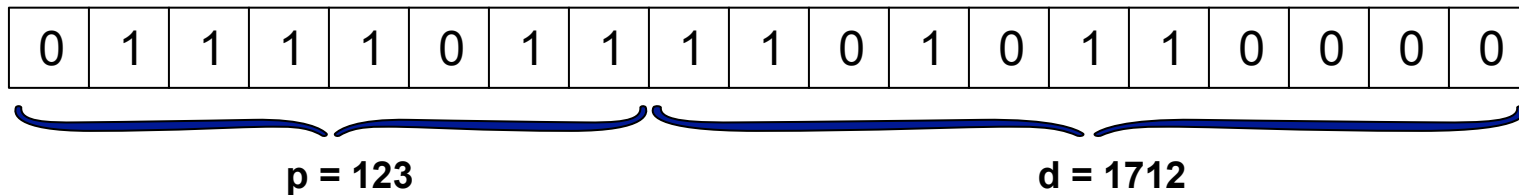
- The virtual/logical page number:  $p$
- The offset within the page:  $d$
- “Read the value at address  $x$ ” becomes “read the value at offset  $d$  in page  $p$ ”

In the above example, how big is each page?

**11 bits  $\rightarrow 2^{11} = 2\text{KiB}$  in a page**

# Virtual Page number

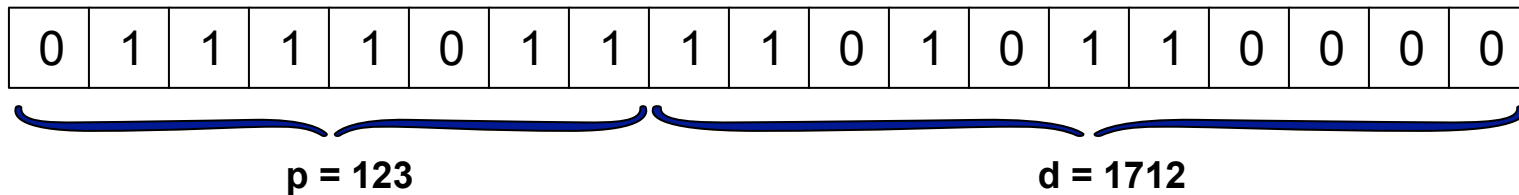
- Virtual addresses issued by the CPU are split into two parts



- The virtual/logical page number:  $p$
  - The offset within the page:  $d$
  - “Read the value at address  $x$ ” becomes “read the value at offset  $d$  in page  $p$ ”
- 
- The process still has the **illusion** of a contiguous address space starting at page 0, continuing at page 1, etc.
  - But in reality (i.e., in the physical RAM), each page is in a memory frame anywhere: We say “**page  $p$  is in frame  $f$** ”

# Virtual Page number

- Virtual addresses issued by the CPU are split into two parts



- The virtual/logical page number:  $p$
- The offset within the page:  $d$
- “Read the value at address  $x$ ” becomes “read the value at offset  $d$  in page  $p$ ”
- The process still has the **illusion** of a contiguous address space starting at page 0, continuing at page 1, etc.
- But in reality (i.e., in the physical RAM), each page is in a memory frame anywhere: We say “**page  $p$  is in frame  $f$** ”
- **Obvious Question:** how do we know in which frame a page is??

# Page-to-Frame Translation

- The Virtual Page Number (VPN) has to be translated to the corresponding Physical Frame Number (PFN)
- This is more sophisticated **address translation** scheme than what we saw in the previous module for contiguous memory allocation
- Remember from the previous slide: instead of “read the value at address x”, a program program does “read the value at offset d in page p”
- Therefore **we need to keep track for each process of the mapping between each of its pages and the physical frame that page is in**
- To this end, each process has a **page table**...

# Page Table Example

- Let's consider a system where the physical memory consists of 8 frames
  - The physical memory has some size, and the OS defines the frame/page size
- Let's say the Kernel fits in one frame

| F# |        |
|----|--------|
| 0  | Kernel |
| 1  | free   |
| 2  | free   |
| 3  | free   |
| 4  | free   |
| 5  | free   |
| 6  | free   |
| 7  | free   |

Physical Memory

# Page Table Example

|        |
|--------|
| Page 0 |
| Page 1 |
| Page 2 |
| Page 3 |

Logical  
Address  
Space

- Let's consider a process whose address space fits in 4 pages
- The OS will place these pages in some of the frames...

| F# |        |
|----|--------|
| 0  | Kernel |
| 1  | free   |
| 2  | free   |
| 3  | free   |
| 4  | free   |
| 5  | free   |
| 6  | free   |
| 7  | free   |

Physical  
Memory

# Page Table Example

|        |
|--------|
| Page 0 |
| Page 1 |
| Page 2 |
| Page 3 |

Logical  
Address  
Space

- Let's consider a process whose address space fits in 4 pages
- The OS will place these pages in some of the frames...
- For instance, as shown on the right
- The OS will maintain a table that maps each page # to a frame #...

| F# |        |
|----|--------|
| 0  | Kernel |
| 1  | Page 0 |
| 2  | free   |
| 3  | Page 2 |
| 4  | Page 1 |
| 5  | free   |
| 6  | free   |
| 7  | Page 3 |

Physical  
Memory



# Page Table Example

|        |
|--------|
| Page 0 |
| Page 1 |
| Page 2 |
| Page 3 |

Logical  
Address  
Space

| Page | Frame |
|------|-------|
| 0    | 1     |
| 1    | 4     |
| 2    | 3     |
| 3    | 7     |

Page  
Table

| F# |        |
|----|--------|
| 0  | Kernel |
| 1  | Page 0 |
| 2  | free   |
| 3  | Page 2 |
| 4  | Page 1 |
| 5  | free   |
| 6  | free   |
| 7  | Page 3 |

Physical  
Memory

# Page Table Example

|        |
|--------|
| Page 0 |
| Page 1 |
| Page 2 |
| Page 3 |

Logical  
Address  
Space

This entry means that  
page 1 is in frame 4

| Page | Frame |
|------|-------|
| 0    | 1     |
| 1    | 4     |
| 2    | 3     |
| 3    | 7     |

Page  
Table

| F# |        |
|----|--------|
| 0  | Kernel |
| 1  | Page 0 |
| 2  | free   |
| 3  | Page 2 |
| 4  | Page 1 |
| 5  | free   |
| 6  | free   |
| 7  | Page 3 |

Physical  
Memory

# Page Size

- The **page size** is defined by the architecture
  - x86-64: 4 KiB, 2 MiB, and 1 GiB
  - ARM: 4 KiB, 64 KiB, and 1 MiB
- The page size in bytes is always a power of 2
- The **pagesize** command gives you the page size on UNIX-like systems
- For instance, on my laptop: 16KiB
- You can easily reconfigure your OS to use a different page size
  - But that page size has to be supported by the hardware
- We'll understand why you may want smaller/bigger pages later...

# Page Size: Address Decomposition

- Say the size of the logical address space is  $2^m$  bytes
- Say a page is  $2^n$  bytes ( $n < m$ ), then...
- The  $n$  low-order bits of a logical address are the offset into the page
  - offset ranges between 0 and  $2^n - 1$ , each one corresponding to a byte in the page
- The remaining  $m - n$  high-order bits are the logical page number
- We already saw this on an example! let's see it on another example again...

# Example

- Physical memory size =  $2^5 = 32$  bytes

# Example

- Physical memory size =  $2^5 = 32$  bytes
- How many bits in a physical address?

|    |  |
|----|--|
| 0  |  |
| 1  |  |
| 2  |  |
| 3  |  |
| 4  |  |
| 5  |  |
| 6  |  |
| 7  |  |
| 8  |  |
| 9  |  |
| 10 |  |
| 11 |  |
| 12 |  |
| 13 |  |
| 14 |  |
| 15 |  |
| 16 |  |
| 17 |  |
| 18 |  |
| 19 |  |
| 20 |  |
| 21 |  |
| 22 |  |
| 23 |  |
| 24 |  |
| 25 |  |
| 26 |  |
| 27 |  |
| 28 |  |
| 29 |  |
| 30 |  |
| 31 |  |

# Example

- Physical memory size =  $2^5 = 32$  bytes
- How many bits in a physical address?
  - How many bits are necessary to address  $2^5$  thingies?

|    |  |
|----|--|
| 0  |  |
| 1  |  |
| 2  |  |
| 3  |  |
| 4  |  |
| 5  |  |
| 6  |  |
| 7  |  |
| 8  |  |
| 9  |  |
| 10 |  |
| 11 |  |
| 12 |  |
| 13 |  |
| 14 |  |
| 15 |  |
| 16 |  |
| 17 |  |
| 18 |  |
| 19 |  |
| 20 |  |
| 21 |  |
| 22 |  |
| 23 |  |
| 24 |  |
| 25 |  |
| 26 |  |
| 27 |  |
| 28 |  |
| 29 |  |
| 30 |  |
| 31 |  |

# Example

- Physical memory size =  $2^5 = 32$  bytes
- How many bits in a physical address?
  - How many bits are necessary to address  $2^5$  thingies?

**5 bits**

|    |  |
|----|--|
| 0  |  |
| 1  |  |
| 2  |  |
| 3  |  |
| 4  |  |
| 5  |  |
| 6  |  |
| 7  |  |
| 8  |  |
| 9  |  |
| 10 |  |
| 11 |  |
| 12 |  |
| 13 |  |
| 14 |  |
| 15 |  |
| 16 |  |
| 17 |  |
| 18 |  |
| 19 |  |
| 20 |  |
| 21 |  |
| 22 |  |
| 23 |  |
| 24 |  |
| 25 |  |
| 26 |  |
| 27 |  |
| 28 |  |
| 29 |  |
| 30 |  |
| 31 |  |



# Example

- Physical memory size =  $2^5 = 32$  bytes
- 5-bit physical addresses

|    |   |       |
|----|---|-------|
| 0  | - | 00000 |
| 1  | - | 00001 |
| 2  | - | 00010 |
| 3  | - | 00011 |
| 4  | - | 00100 |
| 5  | - | 00101 |
| 6  | - | 00110 |
| 7  | - | 00111 |
| 8  | - | 01000 |
| 9  | - | 01001 |
| 10 | - | 01010 |
| 11 | - | 01011 |
| 12 | - | 01100 |
| 13 | - | 01101 |
| 14 | - | 01110 |
| 15 | - | 01111 |
| 16 | - | 10000 |
| 17 | - | 10001 |
| 18 | - | 10010 |
| 19 | - | 10011 |
| 20 | - | 10100 |
| 21 | - | 10101 |
| 22 | - | 10110 |
| 23 | - | 10111 |
| 24 | - | 11000 |
| 25 | - | 11001 |
| 26 | - | 11010 |
| 27 | - | 11011 |
| 28 | - | 11100 |
| 29 | - | 11101 |
| 30 | - | 11110 |
| 31 | - | 11111 |

# Example

- Physical memory size =  $2^5 = 32$  bytes
- 5-bit physical addresses
- Say we pick **frame size = 4 bytes**
  - e.g., Frame #2 contains values at physical addresses 8, 9, 10, 11
- Therefore we also pick **page size = 4 bytes**

|    |   |       |
|----|---|-------|
| 0  | - | 00000 |
| 1  | - | 00001 |
| 2  | - | 00010 |
| 3  | - | 00011 |
| 4  | - | 00100 |
| 5  | - | 00101 |
| 6  | - | 00110 |
| 7  | - | 00111 |
| 8  | - | 01000 |
| 9  | - | 01001 |
| 10 | - | 01010 |
| 11 | - | 01011 |
| 12 | - | 01100 |
| 13 | - | 01101 |
| 14 | - | 01110 |
| 15 | - | 01111 |
| 16 | - | 10000 |
| 17 | - | 10001 |
| 18 | - | 10010 |
| 19 | - | 10011 |
| 20 | - | 10100 |
| 21 | - | 10101 |
| 22 | - | 10110 |
| 23 | - | 10111 |
| 24 | - | 11000 |
| 25 | - | 11001 |
| 26 | - | 11010 |
| 27 | - | 11011 |
| 28 | - | 11100 |
| 29 | - | 11101 |
| 30 | - | 11110 |
| 31 | - | 11111 |

# Example

- Physical memory size =  $2^5 = 32$  bytes
- 5-bit physical addresses
- Say we pick **frame size = 4 bytes**
  - e.g., Frame #2 contains values at physical addresses 8, 9, 10, 11
- Therefore we also pick **page size = 4 bytes**

|            |  |         |
|------------|--|---------|
| 0 - 00000  |  | Frame 0 |
| 1 - 00001  |  |         |
| 2 - 00010  |  |         |
| 3 - 00011  |  |         |
| 4 - 00100  |  | Frame 1 |
| 5 - 00101  |  |         |
| 6 - 00110  |  |         |
| 7 - 00111  |  |         |
| 8 - 01000  |  | Frame 2 |
| 9 - 01001  |  |         |
| 10 - 01010 |  |         |
| 11 - 01011 |  |         |
| 12 - 01100 |  | Frame 3 |
| 13 - 01101 |  |         |
| 14 - 01110 |  |         |
| 15 - 01111 |  |         |
| 16 - 10000 |  | Frame 4 |
| 17 - 10001 |  |         |
| 18 - 10010 |  |         |
| 19 - 10011 |  |         |
| 20 - 10100 |  | Frame 5 |
| 21 - 10101 |  |         |
| 22 - 10110 |  |         |
| 23 - 10111 |  |         |
| 24 - 11000 |  | Frame 6 |
| 25 - 11001 |  |         |
| 26 - 11010 |  |         |
| 27 - 11011 |  |         |
| 28 - 11100 |  | Frame 7 |
| 29 - 11101 |  |         |
| 30 - 11110 |  |         |
| 31 - 11111 |  |         |

# Example

- Physical memory size =  $2^5 = 32$  bytes
- 5-bit physical addresses
- Say we pick **frame size = 4 bytes**
  - e.g., Frame #2 contains values at physical addresses 8, 9, 10, 11
- Therefore we also pick **page size = 4 bytes**
- How many 4-byte frames are there?

$$\frac{2^5 \text{ (bytes)}}{2^2 \text{ (bytes / frame)}} = 2^3 = \mathbf{8 \text{ frames}}$$

|            |  |         |
|------------|--|---------|
| 0 - 00000  |  | Frame 0 |
| 1 - 00001  |  |         |
| 2 - 00010  |  |         |
| 3 - 00011  |  |         |
| 4 - 00100  |  | Frame 1 |
| 5 - 00101  |  |         |
| 6 - 00110  |  |         |
| 7 - 00111  |  |         |
| 8 - 01000  |  | Frame 2 |
| 9 - 01001  |  |         |
| 10 - 01010 |  |         |
| 11 - 01011 |  |         |
| 12 - 01100 |  | Frame 3 |
| 13 - 01101 |  |         |
| 14 - 01110 |  |         |
| 15 - 01111 |  |         |
| 16 - 10000 |  | Frame 4 |
| 17 - 10001 |  |         |
| 18 - 10010 |  |         |
| 19 - 10011 |  |         |
| 20 - 10100 |  | Frame 5 |
| 21 - 10101 |  |         |
| 22 - 10110 |  |         |
| 23 - 10111 |  |         |
| 24 - 11000 |  | Frame 6 |
| 25 - 11001 |  |         |
| 26 - 11010 |  |         |
| 27 - 11011 |  |         |
| 28 - 11100 |  | Frame 7 |
| 29 - 11101 |  |         |
| 30 - 11110 |  |         |
| 31 - 11111 |  |         |

# Example

- Physical memory size =  $2^5 = 32$  bytes
- 5-bit physical addresses
- frame / page size = 4 bytes
- Say we have a process with a 16-byte address space
  - Therefore it has  $16/4 = 4$  pages
- Say its bytes have values a, b, c, ...

|    |   |
|----|---|
| 0  | a |
| 1  | b |
| 2  | c |
| 3  | d |
| 4  | e |
| 5  | f |
| 6  | g |
| 7  | h |
| 8  | i |
| 9  | j |
| 10 | k |
| 11 | l |
| 12 | m |
| 13 | n |
| 14 | o |
| 15 | p |

|            |  |         |
|------------|--|---------|
| 0 - 00000  |  | Frame 0 |
| 1 - 00001  |  |         |
| 2 - 00010  |  |         |
| 3 - 00011  |  |         |
| 4 - 00100  |  | Frame 1 |
| 5 - 00101  |  |         |
| 6 - 00110  |  |         |
| 7 - 00111  |  |         |
| 8 - 01000  |  | Frame 2 |
| 9 - 01001  |  |         |
| 10 - 01010 |  |         |
| 11 - 01011 |  |         |
| 12 - 01100 |  | Frame 3 |
| 13 - 01101 |  |         |
| 14 - 01110 |  |         |
| 15 - 01111 |  |         |
| 16 - 10000 |  | Frame 4 |
| 17 - 10001 |  |         |
| 18 - 10010 |  |         |
| 19 - 10011 |  |         |
| 20 - 10100 |  | Frame 5 |
| 21 - 10101 |  |         |
| 22 - 10110 |  |         |
| 23 - 10111 |  |         |
| 24 - 11000 |  | Frame 6 |
| 25 - 11001 |  |         |
| 26 - 11010 |  |         |
| 27 - 11011 |  |         |
| 28 - 11100 |  | Frame 7 |
| 29 - 11101 |  |         |
| 30 - 11110 |  |         |
| 31 - 11111 |  |         |

# Example

- Physical memory size =  $2^5 = 32$  bytes
- 5-bit physical addresses
- frame / page size = 4 bytes
- How many bits in a virtual address for that process?

|    |   |
|----|---|
| 0  | a |
| 1  | b |
| 2  | c |
| 3  | d |
| 4  | e |
| 5  | f |
| 6  | g |
| 7  | h |
| 8  | i |
| 9  | j |
| 10 | k |
| 11 | l |
| 12 | m |
| 13 | n |
| 14 | o |
| 15 | p |

| p # | f # |
|-----|-----|
| 0   | 5   |
| 1   | 6   |
| 2   | 1   |
| 3   | 2   |

|            |   |         |
|------------|---|---------|
| 0 - 00000  |   | Frame 0 |
| 1 - 00001  |   |         |
| 2 - 00010  |   |         |
| 3 - 00011  |   |         |
| 4 - 00100  | i | Frame 1 |
| 5 - 00101  | j |         |
| 6 - 00110  | k |         |
| 7 - 00111  | l |         |
| 8 - 01000  | m | Frame 2 |
| 9 - 01001  | n |         |
| 10 - 01010 | o |         |
| 11 - 01011 | p |         |
| 12 - 01100 |   | Frame 3 |
| 13 - 01101 |   |         |
| 14 - 01110 |   |         |
| 15 - 01111 |   |         |
| 16 - 10000 |   | Frame 4 |
| 17 - 10001 |   |         |
| 18 - 10010 |   |         |
| 19 - 10011 |   |         |
| 20 - 10100 | a | Frame 5 |
| 21 - 10101 | b |         |
| 22 - 10110 | c |         |
| 23 - 10111 | d |         |
| 24 - 11000 | e | Frame 6 |
| 25 - 11001 | f |         |
| 26 - 11010 | g |         |
| 27 - 11011 | h |         |
| 28 - 11100 |   | Frame 7 |
| 29 - 11101 |   |         |
| 30 - 11110 |   |         |
| 31 - 11111 |   |         |

# Example

- Physical memory size =  $2^5 = 32$  bytes
- 5-bit physical addresses
- frame / page size = 4 bytes
- How many bits in a virtual address for that process?
  - 2-bit page index ( $2^2$  pages)
  - 2-bit offset ( $2^2$  bytes in a page)
  - 4-bit addresses

|    |   |
|----|---|
| 0  | a |
| 1  | b |
| 2  | c |
| 3  | d |
| 4  | e |
| 5  | f |
| 6  | g |
| 7  | h |
| 8  | i |
| 9  | j |
| 10 | k |
| 11 | l |
| 12 | m |
| 13 | n |
| 14 | o |
| 15 | p |

| p # | f # |
|-----|-----|
| 0   | 5   |
| 1   | 6   |
| 2   | 1   |
| 3   | 2   |

|            |   |         |
|------------|---|---------|
| 0 - 00000  |   | Frame 0 |
| 1 - 00001  |   |         |
| 2 - 00010  |   |         |
| 3 - 00011  |   |         |
| 4 - 00100  | i | Frame 1 |
| 5 - 00101  | j |         |
| 6 - 00110  | k |         |
| 7 - 00111  | l |         |
| 8 - 01000  | m | Frame 2 |
| 9 - 01001  | n |         |
| 10 - 01010 | o |         |
| 11 - 01011 | p |         |
| 12 - 01100 |   | Frame 3 |
| 13 - 01101 |   |         |
| 14 - 01110 |   |         |
| 15 - 01111 |   |         |
| 16 - 10000 |   | Frame 4 |
| 17 - 10001 |   |         |
| 18 - 10010 |   |         |
| 19 - 10011 |   |         |
| 20 - 10100 | a | Frame 5 |
| 21 - 10101 | b |         |
| 22 - 10110 | c |         |
| 23 - 10111 | d |         |
| 24 - 11000 | e | Frame 6 |
| 25 - 11001 | f |         |
| 26 - 11010 | g |         |
| 27 - 11011 | h |         |
| 28 - 11100 |   | Frame 7 |
| 29 - 11101 |   |         |
| 30 - 11110 |   |         |
| 31 - 11111 |   |         |

# Example

- What is the **logical** address of byte “g”?
- Logical @ = (page #) \* (page size) + offset
- Page = 1, Offset = 2 (often written 1:2)
- Logical @ =  $1 \times 4 + 2 = 6$

|    |   |
|----|---|
| 0  | a |
| 1  | b |
| 2  | c |
| 3  | d |
| 4  | e |
| 5  | f |
| 6  | g |
| 7  | h |
| 8  | i |
| 9  | j |
| 10 | k |
| 11 | l |
| 12 | m |
| 13 | n |
| 14 | o |
| 15 | p |

| p # | f # |
|-----|-----|
| 0   | 5   |
| 1   | 6   |
| 2   | 1   |
| 3   | 2   |

|            |   |         |
|------------|---|---------|
| 0 - 00000  |   | Frame 0 |
| 1 - 00001  |   |         |
| 2 - 00010  |   |         |
| 3 - 00011  |   |         |
| 4 - 00100  | i | Frame 1 |
| 5 - 00101  | j |         |
| 6 - 00110  | k |         |
| 7 - 00111  | l |         |
| 8 - 01000  | m | Frame 2 |
| 9 - 01001  | n |         |
| 10 - 01010 | o |         |
| 11 - 01011 | p |         |
| 12 - 01100 |   | Frame 3 |
| 13 - 01101 |   |         |
| 14 - 01110 |   |         |
| 15 - 01111 |   |         |
| 16 - 10000 |   | Frame 4 |
| 17 - 10001 |   |         |
| 18 - 10010 |   |         |
| 19 - 10011 |   |         |
| 20 - 10100 | a | Frame 5 |
| 21 - 10101 | b |         |
| 22 - 10110 | c |         |
| 23 - 10111 | d |         |
| 24 - 11000 | e | Frame 6 |
| 25 - 11001 | f |         |
| 26 - 11010 | g |         |
| 27 - 11011 | h |         |
| 28 - 11100 |   | Frame 7 |
| 29 - 11101 |   |         |
| 30 - 11110 |   |         |
| 31 - 11111 |   |         |



# Example

- What is the **physical** address of byte “g”?
- **Physical @ = (frame #) \* (page size) + offset**
- Page = 1 is in Frame 6
- Same Offset = 2
- **Physical @ = 6x4 + 2 = 26**

|    |   |
|----|---|
| 0  | a |
| 1  | b |
| 2  | c |
| 3  | d |
| 4  | e |
| 5  | f |
| 6  | g |
| 7  | h |
| 8  | i |
| 9  | j |
| 10 | k |
| 11 | l |
| 12 | m |
| 13 | n |
| 14 | o |
| 15 | p |

| p # | f # |
|-----|-----|
| 0   | 5   |
| 1   | 6   |
| 2   | 1   |
| 3   | 2   |

|            |   |         |
|------------|---|---------|
| 0 - 00000  |   | Frame 0 |
| 1 - 00001  |   |         |
| 2 - 00010  |   |         |
| 3 - 00011  |   |         |
| 4 - 00100  | i | Frame 1 |
| 5 - 00101  | j |         |
| 6 - 00110  | k |         |
| 7 - 00111  | l |         |
| 8 - 01000  | m | Frame 2 |
| 9 - 01001  | n |         |
| 10 - 01010 | o |         |
| 11 - 01011 | p |         |
| 12 - 01100 |   | Frame 3 |
| 13 - 01101 |   |         |
| 14 - 01110 |   |         |
| 15 - 01111 |   |         |
| 16 - 10000 |   | Frame 4 |
| 17 - 10001 |   |         |
| 18 - 10010 |   |         |
| 19 - 10011 |   |         |
| 20 - 10100 | a | Frame 5 |
| 21 - 10101 | b |         |
| 22 - 10110 | c |         |
| 23 - 10111 | d |         |
| 24 - 11000 | e | Frame 6 |
| 25 - 11001 | f |         |
| 26 - 11010 | g |         |
| 27 - 11011 | h |         |
| 28 - 11100 |   | Frame 7 |
| 29 - 11101 |   |         |
| 30 - 11110 |   |         |
| 31 - 11111 |   |         |

# In-class Exercise (1)

- A computer has 4 GiB of RAM with a page size of 8KiB. Processes have 1 GiB address spaces.
  - How many bits are used for physical addresses
  - How many bits are used for logical addresses
  - How many bits are used for logical page numbers?

# In-class Exercise (1)

- A computer has 4 GiB of RAM with a page size of 8KiB. Processes have 1 GiB address spaces.
  - How many bits are used for physical addresses  
Physical RAM: 4 GiB =  $2^{32}$  bytes  
→ 32-bit physical addresses
  - How many bits are used for logical addresses  
Logical address space: 1 GiB =  $2^{30}$  bytes  
→ 30-bit physical addresses
  - How many bits are used for logical page numbers?  
Page size =  $2^{13}$  bytes  
Number of pages in logical address space:  $2^{30}/2^{13} = 2^{17}$   
→ 17-bit logical page numbers  
(and 13-bit offsets)

## **In-class Exercise (2)**

- Logical addresses are 44-bit, and a process can have up to  $2^{27}$  pages. What is the page size?

## In-class Exercise (2)

- Logical addresses are 44-bit, and a process can have up to  $2^{27}$  pages. What is the page size?

The address space can have up to  $2^{44}$  bytes

There are up to  $2^{27}$  pages

Therefore, a page is  $2^{44} / 2^{27} = 2^{17}$  bytes



## **In-class Exercise (3)**

- On my computer the page size is 16 KiB, and my process' address space is 4GiB.
- How many bits are used for the page number in a logical address?

# In-class Exercise (3)

- On my computer the page size is 16 KiB, and my process' address space is 4GiB.
- How many bits are used for the page number in a logical address?

The address space contains  $2^{32}$  bytes

A page is  $2^{14}$  bytes

Therefore, my address space has  $2^{32}/2^{14} = 2^{18}$  pages

Therefore, we need 18 bits for the page number in a logical address (and we have 14 bits in the offset)

# In-class Exercise (4)

- A computer has 32-bit physical addresses. The logical page number in a logical address is 14-bit. A process can have up to a 2GiB address space
- Let's consider a process with currently a 1GiB address space (i.e., it could get up to another 1GiB during execution).
- What is the page size?
- How many entries in the process' page table currently point to pages that are part of the address space?



# In-class Exercise (4)

- A computer has 32-bit physical addresses. The logical page number in a logical address is 14-bit. A process can have up to a 2GiB address space
- Let's consider a process with currently a 1GiB address space (i.e., it can get up to another 1GiB during execution).
- What is the page size?

Bytes in 2GiB (the max address space):  $2^{31}$

Therefore: 31-bit logical addresses

Therefore:  $31 - 14 = 17$ -bit offsets

Therefore:  $2^{17}$  bytes in a page

Therefore: 128KiB pages

- How many entries in the process' page table currently point to pages that are part of the address space?

The process has a 1GiB =  $2^{30}$ -byte address space

Number of pages in the address space:  $2^{30}/2^{17} = 2^{13}$

Therefore: there are  $2^{13}$  entries in the page table, each pointing to one page

# In-class Exercise (5)

- Logical addresses are 40-bit, and a process can use at most 1/4 of the physical RAM.
  - How big is the RAM?
  - A process has at most  $2^{22}$  pages on this system. How many bits are used for the “offset” part of logical addresses?

# In-class Exercise (5)

- Logical addresses are 40-bit, and a process can use at most 1/4 of the physical RAM.

- How big is the RAM?

With 40-bit logical addresses, an address space is at most  $2^{40}$  bytes

So the RAM is 4 times as big:  $2^{42}$  bytes, which is 4TiB

- A process has at most  $2^{22}$  pages on this system. How many bits are used for the “offset” part of logical addresses?

Since we have  $2^{22}$  pages, 22 bits are used for the page number

Therefore  $40 - 22 = 18$  bits are used for the offset

# In-class Exercise (6)

- Consider a system with 4-byte pages. A process has the following entries in its page table:

| logical | physical |
|---------|----------|
| 0       | 4        |
| 1       | 5        |
| 2       | 30       |

- What is the physical address of the byte with logical address 2?
- What is the physical address of the byte with logical address 9?

# In-class Exercise (6)

- Consider a system with 4-byte pages. A process has the following entries in its page table:

| logical | physical |
|---------|----------|
| 0       | 4        |
| 1       | 5        |
| 2       | 30       |

- What is the physical address of the byte with logical address 2?
- The byte with logical address 2 is the 3rd byte in page 0 (because that page contains the bytes at addresses 0, 1, 2, and 3)
- Page 0, according to the page table is in physical frame 4
- The first byte of physical frame 0 is at physical address  $4 \times 0 = 0$  (the first byte in physical RAM)
- The first byte of physical frame 1 is at physical address  $4 \times 1 = 4$  (the fifth byte in physical RAM)
- ...
- The first byte of physical frame 4 is at physical address  $4 \times 4 = 16$
- The 3rd byte of physical frame is thus at address  $16 + 2$
- Therefore, the byte at logical address 2 is at physical address 18

# In-class Exercise (6)

- Consider a system with 4-byte pages. A process has the following entries in its page table:

| logical | physical |
|---------|----------|
| 0       | 4        |
| 1       | 5        |
| 2       | 30       |

- What is the physical address of the byte with logical address 9?
- The byte with logical address 9 is in page  $9 / 4 = 2$  (integer division)
- Its offset on that page is  $9 \% 4 = 1$
- Page 2 is in frame 30
- Therefore, the byte at logical address 9 is at physical address  $30 \times 4 + 1 = 121$

# Generalization

- If the page size is  $s$
- If the logical address is  $x$
- Then:
  - the logical page number is  $p = \text{floor}(x / s)$
  - the offset is  $o = x \bmod s$
  
- If page  $p$  is in frame  $f$
- Then:
  - logical address  $x$  translates to physical address  $y$   
$$= f * s + o$$

# Sharing Memory Pages

- Time and again we've talked about processes sharing memory
  - Using shared memory IPC
  - With dynamic linking
- It breaks the memory protection abstraction, but it's useful
- Now that we have paging, and that each process has a page table, there is a very simple mechanism to share memory!
- Just create page table entries that point to the same physical frame in different page tables
- Let's see it on a picture...



# Sharing Memory Pages - EASY!

P1 @ space

|          |
|----------|
| Text 1.1 |
| Text 1.2 |
| Text 1.3 |
| Data 1.1 |

P1 page table

|   |    |
|---|----|
| 0 | 3  |
| 1 | 4  |
| 2 | 6  |
| 3 | 10 |

P3 @ space

|          |
|----------|
| Text 3.1 |
| Text 3.2 |
| Text 3.3 |
| Data 3.1 |
| Heap 3.1 |

P3 page table

|   |   |
|---|---|
| 0 | 0 |
| 1 | 5 |
| 2 | 6 |
| 3 | 8 |
| 4 | 2 |

P2 @ space

|          |
|----------|
| Text 2.1 |
| Text 2.2 |
| Text 2.3 |
| Data 2.1 |
| Data 2.2 |
| Heap 2.1 |

P2 page table

|   |   |
|---|---|
| 0 | 3 |
| 1 | 4 |
| 2 | 6 |
| 3 | 1 |
| 4 | 7 |
| 5 | 2 |

Physical Memory

|          |
|----------|
| Text 3.1 |
| Data 2.1 |
| Heap 2.1 |
| Text 1.1 |
| Text 1.2 |
| Text 3.2 |
| Text 1.3 |
| Data 2.2 |
| Data 3.1 |
|          |
| Data 1.1 |

- P1 and P2 share all their text pages (likely invocations of the same program)
- P3 shares one page of its text with P1 and P2 (likely a dynamically linked library, e.g., the code of printf)
- P2 and P3 share one page of heap (likely a shared memory segment)

# Pages Not Allocated (yet)

- So far, we've shown page tables like this:

| Page | Frame |
|------|-------|
| 0    | 1     |
| 1    | 4     |
| 2    | 3     |
| 3    | 7     |

- But in fact, a page table contains entries for all possible pages (up to the maximum allowed number of pages for a process, as defined by the OS

| Page | Frame          |
|------|----------------|
| 0    | 1              |
| 1    | 4              |
| 2    | 3              |
| 3    | 7              |
| 4    | Not used (yet) |
| 5    | Not used (yet) |
| 6    | Not used (yet) |
| 7    | Not used (yet) |

# Valid Bit

- Each page entry is augmented by a **valid bit**
- Set to valid if the process is allowed to access the page (i.e., if the page is in the process address space)
- Set to invalid otherwise
- So page tables look like this:

| Page | Frame | Valid |
|------|-------|-------|
| 0    | 1     | ✓     |
| 1    | 4     | ✓     |
| 2    | 3     | ✓     |
| 3    | 7     | ✓     |
| 4    | xx    | x     |
| 5    | xx    | x     |
| 6    | xx    | x     |
| 7    | xx    | x     |

- If the process references a page whose entry's valid bit is not set, then a trap is generated (and the OS will likely terminate the process)

# What about Fragmentation?

- No external fragmentation!!

- This is of course the HUGE advantage of paging

- Only internal fragmentation

- Worst case: A process address space is  $n$  pages plus 1 byte
    - In this case, we waste 1 page minus 1 byte
  - Average case: Uniform distribution of address space sizes: 50%
    - i.e., on average we waste 1/2 page per process

- Using smaller pages reduces internal fragmentation

- But large pages have advantages:

- Smaller page tables (and less frequent page table lookups)
  - Loading one large page from disk takes less time than loading many small ones

- Typical sizes: 4KiB, 8KiB, 16KiB

- Modern OSes: multiple page sizes supported (Linux: Huge pages; Mac: Superpages; Windows: Large pages) through hardware

# Frames Management

- The OS needs to keep track of the frames
  - Which frames are used (and by which processes?)
  - Which frames are free?
- The OS thus has a data structure: the **free frame list**
- Much simpler than a list of holes with different sizes
  - As done for contiguous memory allocation in the previous “Main Memory” module
- When a process needs a frame, then the OS takes a frame from the free frame list and allocates them to a process

Free frame list = {13, 14, 15, 18, 20}

|  |    |    |    |  |  |    |  |    |  |
|--|----|----|----|--|--|----|--|----|--|
|  | 13 | 14 | 15 |  |  | 18 |  | 20 |  |
|--|----|----|----|--|--|----|--|----|--|

Process creation: P1 needs 4 pages

Free frame list = {15}

|  |      |      |    |  |  |      |  |      |  |
|--|------|------|----|--|--|------|--|------|--|
|  | P1.0 | P1.1 | 15 |  |  | P1.3 |  | P1.2 |  |
|--|------|------|----|--|--|------|--|------|--|

P1's page table

| Page | Frame |
|------|-------|
| 0    | 13    |
| 1    | 14    |
| 2    | 20    |
| 3    | 18    |

# Aside: Memory-Mapped Files

- I/O is very expensive
  - Each access to a file requires a disk seek and a disk access
  - Out of question to read/write bytes one by one to a file
- On-disk address spaces are brought into RAM and virtualized
- Data files can be virtualized **the same way**, i.e., by **mapping** them to memory
- **Memory mapping**
  - Map disk block(s) to a memory frame(s)
  - Initial access is expensive (and generates page faults)
  - Subsequent access is made in memory (and cheap-er)
  - The on-disk file may be updated at a convenient time, upon closing...
  - Memory mapping is performed by dedicated system calls (mmap)
  - Potential concurrency issues: multiple processes can map the same file concurrently
- Let's look at the man page for **mmap**

# Conclusion

- **Paging is great!**

- ☐ No external fragmentation
- ☐ Easy to share pages among processes

- **Mechanisms:**

- ☐ Each process as a page table
- ☐ Each page table entry maps a logical page to a physical frame
- ☐ Each page table entry has a valid bit
- ☐ Address translation is based on the page table
- ☐ The OS manages the list of free frames, and gives out frames to processes

- In the next set of lecture notes, we look at some challenges with paging and how we deal with them...